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RESUMMED QUANTUM GRAVITY

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We present the current status of the a new approach to quantum general relativity based on the exact resummation of its perturbative series as that series was formulated by Feynman. We show that the resummed theory is UV finite and we present some phenomenological applications as well.

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Introduction

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The successful classical generalization of Newton's law of gravity by Einstein in the general theory of relativity is one of the outstanding achievements of 20th century physics. Left over for solution in the 21st century is the problem of the union of general relativity and quantum mechanics. While the superstring theory [1,2] is currently the only accepted solution of this problem with no unresolved theoretical issues, the lack of any experimental verification of superstring theory invites consideration of other approaches. Indeed, the loop-quantum gravity approach is yet another possible solution [3], though it may still have some issues of principle. Accordingly, in Refs. [4–7], we have introduced a new approach to this outstanding problem of quantum general relativity.

Our approach is based on methods well-tested [8,9] in the theory of higher-order radiative corrections in high precision studies at LEP1 and LEP2 and recently extended to high precision LHC physics scenarios [10,11]. We have called the new theory resummed quantum gravity, as we follow Feynman's formulation [12,13] of Einstein's theory as a point particle quantum field theory and, taking a hint from the work of Yennie, Frautschi and Suura [9] in which they discovered that resumming the infrared effects in the electron propagator in QED leads to an improved convergence in the UV for the QED loop corrections that involved that propagator, we show that resumming the large infrared effects in quantum general relativity leads in fact to a UV finite result. This then is a pure union

of the ideas of Bohr and Einstein.

The discussion proceeds as follows. After reviewing Feynman's formulation of Einstein's theory in the next section, we show how resummation renders the theory UV finite in Section 3. Section 4 then presents some phenomenology of the new theory. Section 5 sums up the discussion.

2 Einstein's Theory as Formulated by Feynman

The basic idea of Feynman [12,13] is that quantum general relativity is a point particle field theory where the graviton represents quantum fluctuations about the background metric $\eta_{\mu\nu}$ of space-time: $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$, where $\kappa = \sqrt{8\pi G_N}$ with G_N equal to Newton's constant so that $G_N = 1/M_{Pl}^2$ where $M_{Pl} = 1.22 \times 10^{19} \text{ GeV}$ is the Planck mass. This means that, after we specialize the Lagrangian of the world to the scalar Higgs sector for definiteness (if we show that the Higgs-graviton system is UV finite, the inclusion of the spinning particles will follow without essential complication), then we are led by Feynman [12,13] to consider the Lagrangian

$$\begin{aligned}
 \mathcal{L}(x) = & -\frac{\sqrt{-g}}{2\kappa^2} R + \frac{\sqrt{-g}}{2} \left(g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - m_o^2 \varphi^2 \right) \\
 = & \frac{1}{2} \{ h^{\mu\nu, \lambda} \bar{h}_{\mu\nu, \lambda} - 2\eta^{\mu\mu'} \eta^{\lambda\lambda'} \bar{h}_{\mu\lambda, \lambda'} \eta^{\sigma\sigma'} \\
 & \bar{h}_{\mu'\sigma, \sigma'} \} + \frac{1}{2} \{ \varphi_{, \mu} \varphi^{, \mu} - m_o^2 \varphi^2 \} \\
 & - \kappa h^{\mu\nu} [\overline{\varphi_{, \mu} \varphi_{, \nu}} + \frac{1}{2} m_o^2 \varphi^2 \eta_{\mu\nu}] \\
 & - \kappa^2 [\frac{1}{2} h_{\lambda\rho} \bar{h}^{\rho\lambda} (\varphi_{, \mu} \varphi^{, \mu} - m_o^2 \varphi^2) \\
 & - 2\eta_{\rho\rho'} h^{\mu\rho} \bar{h}^{\rho'\nu} \varphi_{, \mu} \varphi_{, \nu}] + \dots
 \end{aligned} \tag{1}$$

where $\varphi_{,\mu} \equiv \partial_\mu \varphi$ and we have the metric $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$ with $\eta_{\mu\nu} = \text{diag}\{1, -1, -1, -1\}$ and $\bar{y}_{\mu\nu} \equiv \frac{1}{2}(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} y_\rho{}^\rho)$ for any tensor $y_{\mu\nu}$. Feynman has thus formulated Einstein's theory as just another point particle field theory for which he has already worked-out the Feynman rules in Ref. [12, 13].

3 Resummed Quantum Gravity

In Refs. [4–7], we have extended the approach of Yennie, Frautschi and Suura [9] to the Lagrangian in (1) by re-arranging the Feynman series for the 1PI 2-point functions, exactly.

Specifically, as we show in Ref. [4, 7], our exact re-arrangement of the Feynman series for the scalar 1PI 2-point function leads to the result

$$i\Delta'_F(p)|_{\text{resummed}} = \frac{ie^{B''_g(p)}}{(p^2 - m^2 - \Sigma'_s(p) + i\epsilon)} \quad (2)$$

where

$$\Sigma'_s(p) \equiv \sum_{\ell=1}^{\infty} \Sigma'_\ell(p). \quad (3)$$

when $\Sigma'_\ell(p)$ is the corresponding ℓ -loop 1PI 2-point function residual and where the exponent $B''_g(p)$ is given by $(\Delta = k^2 - m^2)$

$$\begin{aligned} B''_g(k) &= -2i\kappa^2 k^4 \frac{\int d^4\ell}{16\pi^4} \frac{1}{\ell^2 - \lambda^2 + i\epsilon} \\ &\quad \frac{1}{(\ell^2 + 2\ell k + \Delta + i\epsilon)^2} \\ &= \frac{\kappa^2 |k^2|}{8\pi^2} \ln \left(\frac{m^2}{m^2 + |k^2|} \right), \end{aligned} \quad (4)$$

where the latter form holds for the UV regime, so that (2) falls faster than any power of $|k^2|$. With this, taken together with its spinning analog representations, for the propagators in the theory, we find that corrections such as those illustrated in Fig. 1 are UV finite when superficially they are $D = +4$, if D is the standard naive power-counting degree of divergence. It can be shown that our resummed theory is entirely UV finite [4]. This is consistent with the more phenomenological analyses in Refs. [14–17], which argue for a similar result following Weinberg's asymptotic safety approach.

We note as well that we know of no contradiction between our UV analysis and the important

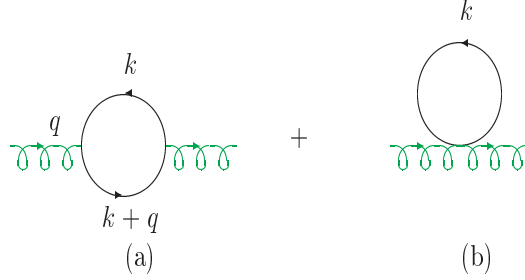


Figure 1. The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

analyses in Refs. [18–20], which deal with the large distance behavior of the theory.

4 Massive Elementary Particles and Black Holes: Final State of Hawking Radiation and Planck Scale Remnants

As we show in Refs. [4–7], when we compute the now UV finite 1-loop contributions to the graviton self-energy, we find the improved Newton potential

$$\Phi_N(r) = -\frac{G_N M}{r} (1 - e^{-ar}), \quad (5)$$

where with

$$a \cong \left(\frac{360\pi M_{Pl}^2}{c_{2,eff}} \right)^{\frac{1}{2}} \quad (6)$$

we have that [7]

$$a \cong 0.210 M_{Pl}. \quad (7)$$

Two consequences of the improved Newton potential are as follows:

4.1 Elementary Particles and Black Holes

A massive point particle of rest mass m has its mass entirely inside of its Schwarzschild radius $r_S = 2m/M_{Pl}^2$ so that classically it should be a black hole. We do not expect this to hold in quantum mechanics. Focusing on the lapse function in the metric class

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2, \quad (8)$$

with

$$f(r) = 1 - \frac{2G(r)m}{r} \quad (9)$$

and $G(r)$, using (5), given by

$$G(r) = G_N(1 - e^{-ar}), \quad (10)$$

we see that the Standard Model massive particles all have the property that $f(r)$ remains positive as r passes through their respective Schwarzschild radii and goes to $r = 0$ – the particle is no longer [5–7] a black hole. Refs. [15, 21] have also found that sub-Planck mass black holes do not exist in quantum field theory.

4.2 Final State of Hawking Radiation – Planck Scale Cosmic Rays

Considering next the evaporation of massive black holes, we first note that in Ref. [15], following Weinberg's [22] asymptotic safety approach as realized by phenomenological exact renormalization group methods, it has been shown that the attendant running of Newton's constant^a leads to the lapse function representation, in the metric class in (8),

$$f(r) = 1 - \frac{2G(r)M}{r} \quad (11)$$

where M is the mass of the black hole and now

$$G(r) \equiv G_{BR}(r) = \frac{G_N r^3}{r^3 + \tilde{\omega} G_N [r + \gamma G_N M]} \quad (12)$$

where γ is a phenomenological parameter [15] satisfying $0 \leq \gamma \leq \frac{9}{2}$ and $\tilde{\omega} = \frac{118}{15\pi}$. It follows [15] as well from (12) that black holes with mass less than a critical mass $M_{cr} \sim M_{Pl}$ have no horizon. Upon joining our result in (9) onto that in (12) at the outermost solution, $r_>$, of the equation

$$G_{BR}(r) = G_N(1 - e^{-ar}), \quad (13)$$

we find the following for the final state of the Hawking process for an originally very massive black hole: for $r < r_>$, we use our result in (9) for $G(r)$ and for $r > r_>$ we use $G_{BR}(r)$ for $G(r)$ after the originally massive black hole has Hawking radiated down to the appropriate scale. For the self-consistent value $\gamma = 0$ and $0.2 = \Omega \equiv \frac{\tilde{\omega}}{G_N M^2} =$

$\frac{\tilde{\omega} M_{Pl}^2}{M^2}$ for definiteness we find that the inner horizon found in Ref. [15] moves to negative values of r and that the outer horizon moves to $r = 0$, so that the entire mass of the originally very massive black hole radiates away until a completely accessible Planck scale remnant of mass $M'_{cr} = 2.38 M_{Pl}$ is left: It would be expected to decay into n -body final states, $n = 2, 3, \dots$, leading in general to Planck scale cosmic rays [6, 7]. The data in Ref. [25, 26] are not inconsistent with this conclusion, which also agrees with recent results by Hawking [27].

5 Conclusions

In this paper we have presented a new paradigm in the history of point particle field theory: a UV finite theory of the union of quantum mechanics and general relativity. It holds promise to be a solution to most of the outstanding problems in the union of the ideas of Bohr and Einstein. More importantly, it is clear evidence that quantum mechanics, while not necessarily the ultimate theory, is not an incomplete theory. This work was supported in part by US DOE grant DE-FG02-05ER41399 and by NATO grant PST.CLG.980342. We thank Prof. S. Jadach for useful discussions.

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